Dear Family,

The next unit in your child's mathematics class this year is ***Frogs, Fleas, and Painted Cubes: Quadratic Relationships***. In an earlier unit, *Thinking With Mathematical Models*, students studied the use of linear models and investigated examples of inverse variation. In the *Growing, Growing, Growing* unit, students explored exponential models. In *Frogs, Fleas, and Painted Cubes*, the focus switches to a nonlinear polynomial relationship: the second-degree polynomial or the *quadratic* function.

UNIT GOALS

Students will learn to recognize quadratic patterns of change in tables and graphs, and they will learn to write equations to represent those patterns. They will compare and contrast quadratic patterns of change with those of linear and exponential patterns of change, which they have already studied in depth.

Quadratic relationships are encountered in such fields as business, sports, engineering, and economics. We are dealing with quadratic relationships, for example, when we study how the height of a ball—or jumping flea—changes over time. A quadratic graph, called a *parabola*, is shaped like either a U or an upside-down U.

HELPING WITH HOMEWORK

You can help with your child's homework and encourage sound mathematical habits as your child studies this unit by asking questions such as:

- How can I recognize if the relationship among variables in a situation is a quadratic function?
- What equation would represent a quadratic relationship in the table, graph, or problem context relating the variables?
- How could I answer the questions of the situations by studying a table, graph, or equation of the quadratic relationship?

In your child's notebook, you can find worked-out examples from problems done in class, notes on the mathematics of the unit, and descriptions of the vocabulary words.

HAVING CONVERSATIONS ABOUT THE MATHEMATICS IN *FROGS, FLEAS, AND PAINTED CUBES*

You can help your child with his or her work for this unit in several ways:

- Talk with your child about the situations that are presented in the unit.
- Look over your child's homework and make sure that all questions are answered and that explanations are clear.
- Have your child pick a question that was interesting to him or her and explain it to you.

A few important mathematical ideas that your child will learn in *Frogs, Fleas, and Painted Cubes* are given on the back. As always, if you have any questions or concerns about this unit or your child's progress in class, please feel free to call.

Sincerely,

Important Concepts	Examples

Representing Quadratic Patterns of Change With Tables

In linear relationships, the *first differences* of successive values are constant, indicating a constant rate of change. In quadratic relationships, first differences are not constant, but *second differences* are. The first difference is the rate at which y is changing with respect to x. The second difference indicates the rate at which *that rate* is changing. If the second differences are all the same, then the relationship is quadratic.

$y = 6(x - 2)^2$

x	y
0	24
1	6
2	0
3	6
4	24
5	54

First Differences

- $6 - 24 = {}^-18$
- $0 - 6 = {}^-6$
- $6 - 0 = 6$
- $24 - 6 = 18$
- $54 - 24 = 30$

Second Differences

- ${}^-6 - ({}^-18) = 12$
- $0 - ({}^-6) = 12$
- $18 - 6 = 12$
- $30 - 18 = 12$

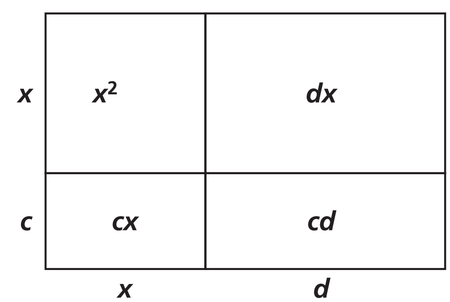
The second differences are all 12, which indicates that the table represents a quadratic relationship.

Representing Quadratic Functions With Equations

Traditionally, quadratic relationships are defined as relationships that have equations fitting the form $y = ax^2 + bx + c$, in which a, b, and c are constants, and $a \neq 0$. This form of the equation is called the *expanded form*. The emphasis is on observing that the equations contain an independent variable raised to the second power. It is also important to understand the *factored form* of such equations.

Many quadratic equations can also be defined as functions whose y-value is equal to the product of two linear factors—the form $y = (ax + c)(bx + d)$, where $a \neq 0$ and $b \neq 0$. The power of this form is that it relates quadratic polynomials as products of linear factors.

The area of the rectangle below can be thought of as the product of two linear expressions, as the result of multiplying the width by the length, or as the sum of the area of the subparts of the rectangle.



$A = (x + c)(x + d)$ factored form

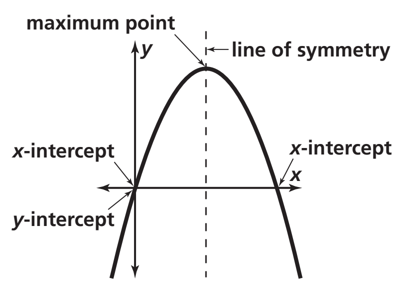
$A = x^2 + cx + dx + d$ expanded form

Representing Quadratic Patterns of Change With Graphs

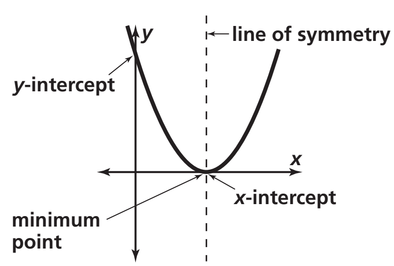
The values in the equation affect the shape, orientation, and location of the quadratic graph, a parabolic curve.

If the coefficient of the x^2 term is positive, the curve opens upward and has a minimum point. If negative, the curve opens downward and has a maximum point.

The maximum or minimum point of a quadratic graph **(parabola)** is called the **vertex**. The vertex lies on the vertical *line of symmetry* that separates the parabola into halves that are mirror images. The vertex is located halfway between the ***x*-intercepts**, if the x-intercepts exist. The x-intercepts are mirror images of each other. The ***y*-intercept** is where the parabola crosses the y-axis.



$y = -2x^2 + 8x$



$y = x^2 - 8x + 16$

On the **CMP Parent Web Site,** you can learn more about the mathematical goals of each unit, see an illustrated vocabulary list, and examine solutions of selected ACE problems. **http://PHSchool.com/cmp2parents**